1 Prerequisites

Students entering the MSc course are supposed to be familar with a number of topics, which would normally be covered in an undergraduate degree. They fall into two categories, namely elementary mathematics and elementary physics. It would be a very good idea to recapitulate these topics before starting the course, because familiarity with these concepts will be assumed.

1. **Elementary Mathematics:**
   - **Linear algebra:** Scalar products of vectors, vector products. Matrices, multiplication, inversion, determinants, eigenvalues and eigenvectors diagonalisation by change of basis, orthogonal and unitary matrices, diagonalisation of symmetric matrices by orthogonal matrices.
   - **Complex numbers:** addition, multiplication, complex conjugate. Complex functions, differentiation, Cauchy-Riemann equations. Elementary functions: exp(z), log(z), cos(z), cosh(z), etc. Cuts and branches. Cauchy's integral formula. Evaluation of integrals (and series) by calculus of residues. Analytic continuation.
   - **Elementary group theory:** axioms of groups, examples: GL(n), SL(n), O(n).
   - **Functions of several variables:** Continuity. Partial differentiation. Chain rule. Taylor polynomial in two variables. The gradient and directional derivatives of a function. Line integrals of functions and vector fields. The divergence and the curl of a vector field. Index notation, summation convention. Epsilon(ijk) and delta(ij). Local and global extrema of functions. Extrema of functions with constraints.
   - **Multiple Integration:** iterated sums, double and triple integrals by repeated integration, volume enclosed by surface, Jacobians and change of variables. Line, surface and volume integrals. Stokes' and divergence theorems.
   - **PDEs:** Solution of Poisson's and Laplace's Equations: Uniqueness of solution of Laplace's and Poisson's equations. General solution of Poisson's equation. Green's function. Simple examples of solution of Laplace's equation by separation of variables.
   - **Fourier Series:** Orthogonal functions and Fourier series. Convergence, periodic extension, sine and cosine series, half-range expansion. Fourier transform and inverse, convolution theorem. Solution of heat equation on n-dimensional Euclidean space using Fourier transform and construction of heat kernel. Connection between heat kernel and Green's function. Laplace transform.
2. **Elementary physics:**
   - **Mechanics:** Frames of reference, Newton's laws in vector form, forces, mass, momentum, gravitational force, projectiles, Lorentz force and charged particles inconstant electromagnetic fields. Concepts of energy and angular momentum. Simple harmonic motion and oscillations about a stable equilibrium. Damped

oscillations and resonance. Central forces and the use of energy and angular momentum to study planetary motion. Waves and strings, including the derivation of the wave equation for small amplitude vibrations and its solution by separation of variables. Energy, energy density, energy carried by wave.

- **Lagrangian Mechanics:** Lagrangian and Hamiltonian Dynamics: Hamilton's principle in dynamics. Generalised coordinates and momenta. Derivation of Lagrange's equations. Generalised forces. Conservative forces. Ignorable coordinates and conservation laws. Noether's theorem. Systems with constraints Hamilton's equations. Formulation in terms of Poisson brackets. Small oscillations of systems of particles: Positions of equilibrium and stability. Normal modes of oscillation and normal coordinates. Stationary properties of frequencies of systems with constraints. Basic variational calculus.
- **Special Relativity:** Inertial frames. Speed of light. Events. Spacetime. Time dilation and length contraction. Lorentz transformations. Standard Lorentz boosts. Composition of velocities. Doppler effect. Group structure of standard Lorentz boosts. Four vectors: Minkowski spacetime. Lorentz and Poincare groups. Worldline and light cone. Causality. Proper time, velocity, acceleration. Space-time vectors and tensors. Mass-energy equivalence. Einstein's relation. Zero-mass particles. Systems of free particles: Conservation of 4-momentum. Centre-of-mass frame. Collision processes.
- **Classical Electromagnetism:** Maxwell's equations with sources. Potentials and gauge invariance. Wave equation. Energy and momentum conservation (including energy density, Poynting vector, stress tensor). Plane waves. Polarisation. Retarded potentials. Special Relativistic Formulation of Electromagnetism: invariances of Maxwell's equations, the equations with microscopic sources expressed as tensor equations in Minkowski spacetime. Relativistic equation of motion for a charged particle in an external electromagnetic field.
- **Quantum Mechanics:** Photo-electric effect, atomic spectra, wave-particle duality, uncertainty principle.Vectors in Hilbert space, linear operators, hermitian operators, eigenvalues, complete sets, expectation values, commutation relations, observables and commuting operators, symmetries and spectra. Dirac notation. Schroedinger and Heisenberg pictures. Position operator, Harmonic Oscillator, creation-annihilation operators. Angular momentum, spin representations. Time–dependent and time–independent Schroedinger equation. Probability interpretation. Currents. Plane-waves, spreading of a wave-packet. Perturbation theory, interaction picture.

In addition, there are a number of things which you may have studied, but if you haven't it would be a good idea to read about before you come because these will be developed in greater depth at the beginning of the course:

3. <u>**Advanced topics:**</u>

- **General Relativity:** gravity as geometry, equivalence principle. Differential Manifolds: spacetime as a manifold, coordinates and coordinate transformations, tangent vectors, tensors under general co-ordinate transformations.

Metric: distance relationships, light cones. Covariant Derivative: parallel transport, connection coefficients, differentiating tensors, metric connection, geodesics. Curvature: Riemann tensor, characterisation of flat space, parallel transport around closed curves, commutation formulae, Bianchi identity, Einstein tensor, geodesic deviation. Einstein's equations, linearized theory and Newtonian limit, Einstein–Hilbert action.

- **Quantum Field Theory:** Quantisation of free scalar fields: Multi-particle quantum mechanics, canonical quantisation of free scalar fields, Fock space, anti-particles, propagators, causality. Evolution operators, perturbative expansion, Wick's theorem, Feynman diagrams in position and momentum space, LSZ reduction, scattering matrix, cross sections.
- **Statistical Mechanics:** Thermodynamics: Thermal equlibrium, the laws of thermodynamics. Equations of state, ideal gas law. Classical statistical mechanics: Statistical basis of thermodynamics: microstates, macrostates and the thermodynamics limit. Ideal gas. Gibbs paradox and entropy. Microcanonical, canonical and grand-canonical ensembles. Distributions and identical particles: Maxwell-Boltzmann distribution. Bose and Fermi distributions.